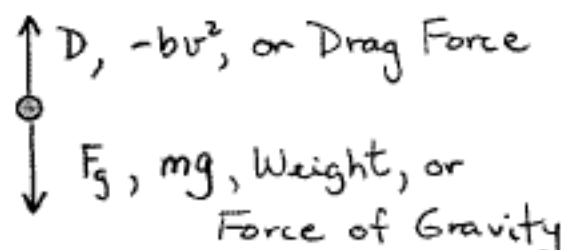


# 2000 Physics C Solutions

## Distribution of points

Mech. 2 (15 points)

(a) 3 points



For a vector arrow pointing downward

1 point

For a vector arrow pointing upward

1 point

For correct force labels on both vectors

1 point

For any extra vectors drawn, deduct 1 point

(b) 3 points

For indicating that the acceleration decreases

1 point

For a correct explanation that includes a correct mention of forces.

2 points

For example, as the ball approaches terminal speed, the velocity increases, so the drag force increases and gets closer in magnitude to the gravitational force. The resultant force, which is the difference between the gravitational and drag forces, gets smaller, and since it is proportional to the acceleration, the acceleration decreases.

Partial credit of 1 point given for only a statement including a basic definition of terminal velocity (e.g., at terminal velocity  $v = \text{constant}$ , so  $a$  must decrease from  $9.8 \text{ m/s}^2$  to zero)

(c) 2 points

For an expression for the resultant force on the ball

1 point

$$F = mg - bv^2$$

Since  $F = ma = m \frac{dv}{dt}$ , then  $m \frac{dv}{dt} = mg - bv^2$

For a correct differential equation

1 point

$$\frac{dv}{dt} = g - \frac{b}{m} v^2$$

Students did not need to use the convention  $+$  and  $-$  for up and down, respectively, but they did have to be consistent in their sign notation for credit. The integral form of the differential equation was also acceptable.

2000 Physics C Solutions

Distribution  
of points

Mech. 2 (continued)

(d) 3 points

For recognition that acceleration is zero at terminal speed

1 point

For setting the drag force equal to the gravitational force

1 point

$$mg = bv_t^2$$

For a correct solution for  $v_t$

1 point

$$v_t = \sqrt{\frac{mg}{b}}$$

Full credit also given for writing answer only with no other work shown

(e) 4 points

For a correct statement of work-energy, recognizing that the energy dissipated by the drag force is equal to the initial energy minus the final energy

1 point

For correct recognition of both initial potential energy  $mgh$  and final kinetic energy

1 point

$$\frac{1}{2}mv_t^2$$

$$\Delta E = mgh - \frac{1}{2}mv_t^2$$

For correct substitution of  $v_t$  from part (d)

1 point

$$\Delta E = mgh - \frac{1}{2}m\left(\frac{mg}{b}\right)$$

For correct answer

1 point

$$\Delta E = mg\left(h - \frac{m}{2b}\right)$$

Alternate partial solution (for maximum credit of 2 points)

(Alternate points)

For a correct integral for work

(1 point)

$$W = \int P dt \quad \text{OR} \quad W = \int F dx$$

For correct substitutions for  $P$  or  $F$

(1 point)

$$W = \int bv^3 dt \quad \text{OR} \quad W = \int bv^2 dx \quad \text{OR} \quad W = \int kv^2 dx$$

**AP<sup>®</sup> PHYSICS C MECHANICS  
2005 SCORING GUIDELINES**

**Question 1**

**15 points total**

**Distribution  
of points**

(a) 2 points

For indicating that the magnitude of the acceleration decreases as the ball moves upward  
 For a correct, reasonable justification  
 For example: Since velocity is upward, air resistance is downward, in the same direction as gravity. The velocity will decrease, causing the force of air resistance to decrease. Therefore, the net force and thus the total acceleration both decrease.

1 point  
1 point

(b) 3 points

For showing the expression  $a = \frac{dv}{dt}$

1 point

For any clear indication that the forces of air resistance and gravity are in the same direction, such as by showing an equation or a free-body diagram

1 point

$$F_{\text{net}} = -Mg - kv$$

For a correct differential equation with the correct signs

1 point

$$M \frac{dv}{dt} = -Mg - kv$$

(c) 3 points

For recognizing that at terminal speed  $F_{\text{net}} = 0$

1 point

For any clear indication that the forces of air resistance and gravity are now in opposite directions, such as by showing an equation or a free-body diagram

1 point

$$F_{\text{net}} = -Mg + kv$$

$$0 = -Mg + kv_T$$

For a correct expression for the terminal speed

1 point

$$v_T = Mg/k$$

(d) 2 points

For indicating that it takes longer for the ball to fall

1 point

For a correct, reasonable justification

1 point

For example: The ball loses mechanical energy on the way up and on the way down.

This means the average speed is greater on the way up than on the way down.

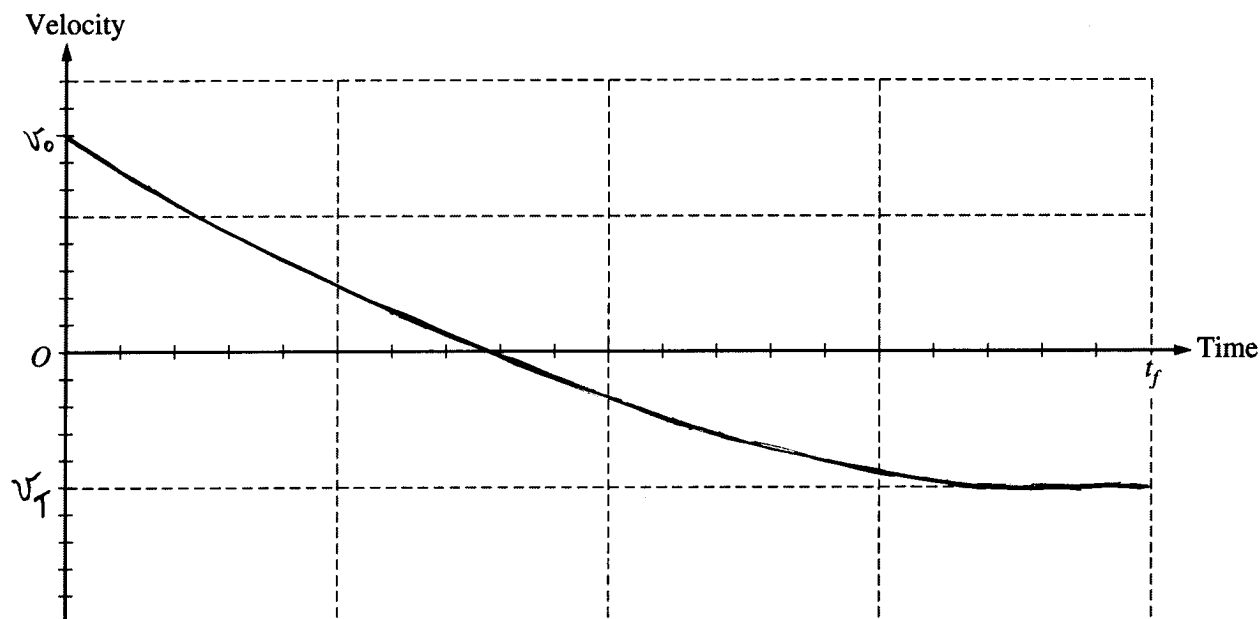
Since the distance traveled is the same, the time must be longer on the way down.

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2005 SCORING GUIDELINES

Question 1 (continued)

Distribution  
of points

(e) 5 points



For an exponentially decreasing curve beginning with positive initial velocity  $v_0$ , crossing the time axis at  $t$  less than  $t_f/2$ , and having the final speed less than the initial speed 5 points

One point partial credit was awarded for each of the following curve characteristics.

For showing that when  $v = 0$ , the curve is differentiable (i.e., no discontinuity in slope) and has a negative slope

For showing the curve to be concave upward both for when the ball is rising and when the ball is falling

For showing time intervals for when the ball is rising and when the ball is falling that are consistent with the answer to part (d)

For showing that the final velocity is negative and that the speed at  $t_f$  is less than  $v_0$

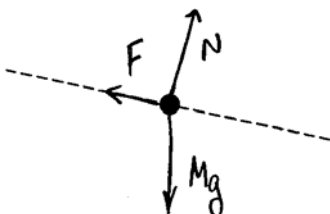
# AP<sup>®</sup> PHYSICS C: MECHANICS 2008 SCORING GUIDELINES

## Question 1

15 points total

Distribution  
of points

(a) 3 points



- For a correctly drawn and labeled weight vector, originating on the dot and with an arrowhead (Alternatively, correctly drawn and labeled components instead of the total weight vector was acceptable.) 1 point
- For a correctly drawn and labeled normal force vector, originating on the dot and with an arrowhead 1 point
- For a correctly drawn and labeled drag-force vector, originating on the dot and with an arrowhead 1 point
- One point was deducted if there were any extra vectors on the point, including components drawn with arrowheads.

(b) 4 points

- For any expression of  $F = Ma$  or any dimensionally correct application of  $F = Ma$  1 point
- For correctly expressing the component of the weight parallel to the plane as  $Mg \sin \theta$  1 point
- For correctly expressing the drag force as  $-kv$  1 point
- $Ma = Mg \sin \theta - bv$
- For a dimensionally correct differential equation, including  $dv/dt$  and expressions for the drag force and the component of the weight parallel to the plane 1 point
- $M \frac{dv}{dt} = Mg \sin \theta - bv$
- One point was deducted if the algebraic signs of the weight component and the drag force were not opposite somewhere in the solution, OR if only one of these two terms was included.

(c) 2 points

- For an indication that  $F_{\text{net}} = 0$ ,  $a = 0$ , or the parallel component of the weight =  $bv_T$  1 point
- $0 = Mg \sin \theta - bv_T$
- $bv_T = Mg \sin \theta$
- For the correct expression for the terminal velocity (or one consistent with part (b)) 1 point
- $v_T = Mg \sin \theta / b$

**AP<sup>®</sup> PHYSICS C: MECHANICS**  
**2008 SCORING GUIDELINES**

**Question 1 (continued)**

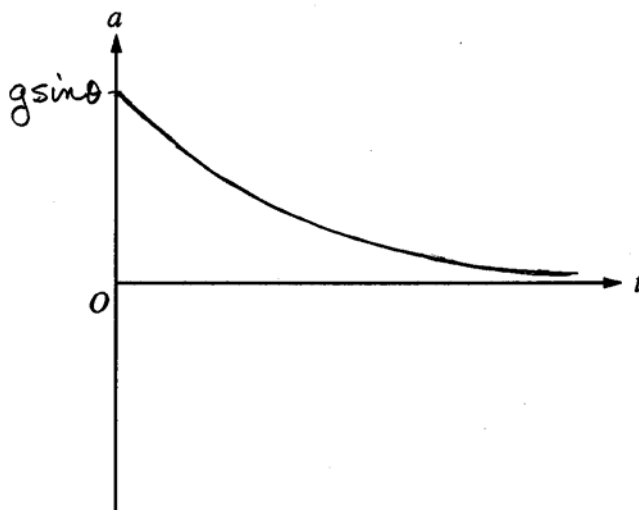
|                                                                                                                                                                           | <b>Distribution<br/>of points</b> |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------|
| (d) 3 points                                                                                                                                                              |                                   |
| For taking the differential equation from part (b) and correctly separating the variables in preparation for integration (definite or indefinite integral)                | 1 point                           |
| $M \frac{dv}{dt} = Mg \sin \theta - bv$ $\frac{dv}{Mg \sin \theta - bv} = \frac{dt}{M}$                                                                                   |                                   |
| For correct integration of both sides of equation                                                                                                                         | 1 point                           |
| For example, using a method involving an indefinite integral                                                                                                              |                                   |
| Letting $u = Mg \sin \theta - bv$ , so $du = -b dv$                                                                                                                       |                                   |
| $-\frac{1}{b} \frac{du}{u} = \frac{dt}{M}$ $\int \frac{du}{u} = -\frac{b}{M} \int dt$ $\ln u = -\frac{b}{M}t + \ln C$ $u = Ce^{-bt/M}$ $Mg \sin \theta - bv = Ce^{-bt/M}$ |                                   |
| Using $v = 0$ at $t = 0$                                                                                                                                                  |                                   |
| $Mg \sin \theta = C$                                                                                                                                                      |                                   |
| $Mg \sin \theta - bv = Mg \sin \theta e^{-bt/M}$                                                                                                                          |                                   |
| $-bv = Mg \sin \theta e^{-bt/M} - Mg \sin \theta$                                                                                                                         |                                   |
| For a correct final expression for $v(t)$                                                                                                                                 | 1 point                           |
| $v = (Mg \sin \theta / b)(1 - e^{-bt/M})$                                                                                                                                 |                                   |

**AP<sup>®</sup> PHYSICS C: MECHANICS**  
**2008 SCORING GUIDELINES**

**Question 1 (continued)**

**Distribution  
of points**

(e) 3 points



For the correct initial value of  $a$  (or a value consistent with part (b))

1 point

For a negatively sloped curve, concave up

1 point

For a curve asymptotic to the  $t$  axis

1 point

(This point was awarded even if the curve was not otherwise correct.)

**AP<sup>®</sup> PHYSICS C: MECHANICS  
2011 SCORING GUIDELINES**

**Question 2 (continued)**

**Distribution  
of points**

(e)

i. 2 points

$$\Sigma F = ma$$

For substituting the braking force into Newton's second law as the net force

1 point

For substituting the time derivative of velocity for the acceleration

1 point

$$-ku = M(du/dt)$$

ii. 2 points

For separating the variables and integrating

1 point

$$du/u = -(k/M) dt$$

$$\int_{u_D}^u du/u = -(k/M) \int_0^t dt$$

$$\ln u \Big|_{u_D}^u = -(k/M)t$$

$$\ln u - \ln u_D = \ln(u/u_D) = -(k/M)t$$

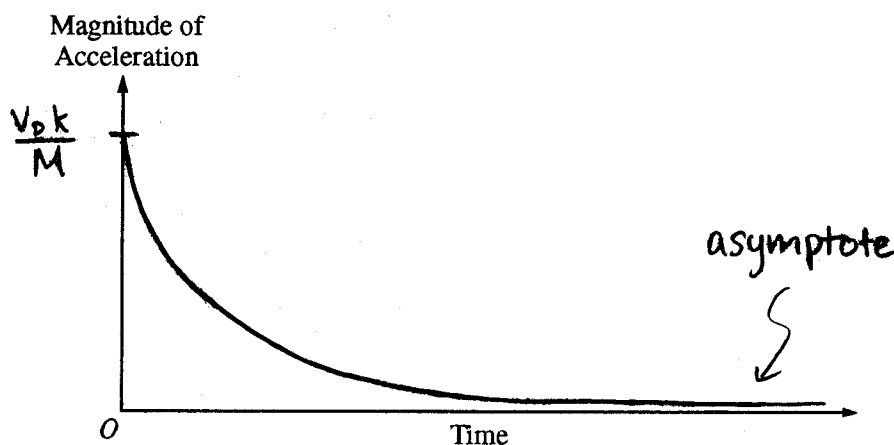
$$u/u_D = e^{-kt/M}$$

For a correct expression for the velocity as a function of time

1 point

$$u = u_D e^{-kt/M}$$

iii. 3 points



Taking the derivative of the equation for  $u$  from part (e) ii

$$a = du/dt = d(u_D e^{-kt/M})/dt = -(k/M)u_D e^{-kt/M}$$

$$\text{At } t = 0, a = -ku_D/M$$

For a graph with a finite intercept on the vertical axis

1 point

For a graph that is concave upward and asymptotic to zero

1 point

For labeling the initial acceleration with the correct value

1 point

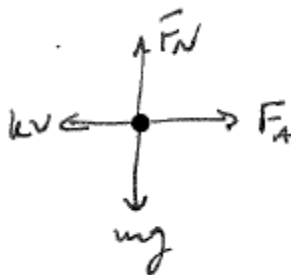
**AP<sup>®</sup> PHYSICS C: MECHANICS  
2013 SCORING GUIDELINES**

**Question 2**

**15 points total**

**Distribution  
of points**

(a) 4 points



For correctly showing and labeling the applied force directed to the right  
 For correctly showing and labeling the downward gravitational force  
 For correctly showing and labeling the upward normal force  
 For correctly showing and labeling the drag force directed to the left  
 One earned point was deducted for having any extraneous vectors

1 point  
 1 point  
 1 point  
 1 point

(b) 2 points

$$F_{net} = ma$$

For the correct substitution into Newton's second law

$$F_A - kv = ma$$

For a correct differential equation

$$F_A - kv = m \frac{dv}{dt}$$

1 point  
 1 point

(c) 1 point

Set  $\frac{dv}{dt} = 0$  in the equation from part (b)

$$F_A - kv = 0$$

For the correct expression for the terminal velocity

$$v_T = \frac{F_A}{k}$$

1 point

**AP<sup>®</sup> PHYSICS C: MECHANICS**  
**2013 SCORING GUIDELINES**

**Question 2 (continued)**

**Distribution  
of points**

(d) 5 points

Use the differential equation from part (b)

$$F_A - kv = m \frac{dv}{dt}$$

For demonstrating separation of variables

1 point

$$\frac{1}{m} dt = \frac{1}{F_A - kv} dv$$

For demonstrating that the equation must be integrated

1 point

$$\int \frac{1}{m} dt = \int \frac{1}{F_A - kv} dv$$

For demonstrating substitution using initial and final values (or evaluating the constant of integration using the boundary conditions)

1 point

$$\int_0^t \frac{1}{m} dt = \int_0^{v(t)} \frac{1}{F_A - kv} dv$$

$$\left[ \frac{t}{m} \right]_0^t = -\frac{1}{k} [\ln(F_A - kv)]_0^{v(t)}$$

For attempting to solve for  $v(t)$

1 point

$$-\frac{kt}{m} = \ln\left(\frac{F_A - kv(t)}{F_A}\right)$$

$$e^{-kt/m} = \frac{F_A - kv(t)}{F_A} = 1 - \frac{kv(t)}{F_A}$$

$$\frac{kv(t)}{F_A} = 1 - e^{-kt/m}$$

For a correct answer

1 point

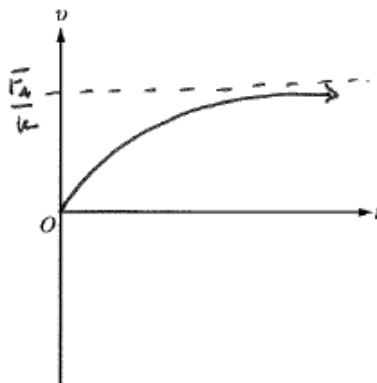
$$v(t) = \frac{F_A}{k} (1 - e^{-kt/m})$$

**AP<sup>®</sup> PHYSICS C: MECHANICS**  
**2013 SCORING GUIDELINES**

**Question 2 (continued)**

**Distribution  
of points**

(e) 3 points



For a graph that begins at the origin, with a non-negative slope everywhere,  
and is concave downward

1 point

For a graph with a horizontal asymptote

1 point

For the correct label of the expression for the asymptote or maximum on the  
vertical axis

1 point